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LINEARIZED THEORY OF FOIL IN SUBSONIC FLOW, (U)
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by

G. A. Dombrovskiy, V. S. Tkalich

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А а	*А а*	A, a	Р р	*Р р*	R, r
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В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
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Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

LINEARIZED THEORY OF FOIL IN SUBSONIC FLOW

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The main results of the linearized theory of a subsonic gas flow past a foil is contained in the formula

$$C_p = \frac{C_p^0}{\sqrt{1 - M_\infty^2}}, \quad (0.1)$$

where C_p is the pressure coefficient at points on the foil in the gas flow, C_p^o - pressure coefficient at corresponding points on the same foil in a flow of an incompressible fluid, M_∞ - value of the Mach

number at an infinitely distant point in the gas flow.

As our first approximation formula (0.1) is correct for the arbitrary relationship between density ρ and pressure p. In the present study we demonstrate that for each $M_{\infty} < 1$ we can find a dependence between ρ and p at which formula (0.1) becomes precise. Because of this result it is possible to approach evaluation of the accuracy of the linearized theory differently and to compose a more graphic linearized theory with methods based on approximations of the indicated connection between density and pressure and, primarily, by the well known approximate method of Chaplygin.

§ 1

Let us proceed from a linear system in hodograph variables, reduced to the symmetrical form of

$$\frac{\partial\varphi}{\partial\theta}=\sqrt{K}\frac{\partial\psi}{\partial s}, \quad \frac{\partial\varphi}{\partial s}=-\sqrt{K}\frac{\partial\psi}{\partial\theta}, \tag{1.1}$$

where ϕ is the velocity potential, ψ - current function, θ - angle of slope of velocity vector, s (v) - function of velocity modulus v, K(s) - Chaplygin function [1, 3].

In his time R. Sauer [2] examined the case of

(1.2) $$K = n^4\sigma^4, \quad \sigma = s - s_0,$$

where n and s_0 are actual arbitrary constants.

There was a similar study of this case in monograph [1]. According to [1], the general solutions of system (1.1) with the Chaplygin function (1.2) and corresponding formulas for velocity v, density ρ, and pressure p are written:

(1.3) $$\varphi = n^2 \Re e\left[\sigma F(\mathfrak{z}) - \int F(\mathfrak{z})\, d\mathfrak{z}\right], \quad \psi = \frac{1}{\sigma} \Im m\, F(\mathfrak{z}),$$

where $F(\mathfrak{z})$ represents the arbitrary analytical function of complex variable $\mathfrak{z} = s - i\theta$.

(1.4) $$\begin{cases} v(\sigma) = \dfrac{B\sigma}{A\,\mathrm{sh}\,\sigma + \mathrm{ch}\,\sigma}, \\ \rho(\sigma) = \dfrac{A\,\mathrm{sh}\,\sigma + \mathrm{ch}\,\sigma}{n^2\sigma\,[A\,(\mathrm{sh}\,\sigma - \sigma\,\mathrm{ch}\,\sigma) + (\mathrm{ch}\,\sigma - \sigma\,\mathrm{sh}\,\sigma)]}, \\ p(\sigma) = \dfrac{B^2}{n^2 A}\,\dfrac{1}{A\,\mathrm{th}\,\sigma + 1} + C, \end{cases}$$

where A, B, C are arbitrary constants.

Also interesting is the dependence M = M(σ) which can be determined using (1.2) and the second of the relationships of (1.4)

according to formula [3]

$$M=\sqrt{1-K\rho^{3}}. \tag{1.5}$$

The transition to the physical plane of the gas flow Z = X + iY is achieved through formula ([1], page 63)

$$Z=\frac{n^2}{2}\left\{\frac{e^{i\theta}}{v}\left[F(\mathfrak{z})-\overline{F(\mathfrak{z})}\right]-\right.$$
$$\left.-\frac{1}{B}\left[(A-1)\int\frac{V_0}{e^{\mathfrak{z}}}\frac{dF}{d\mathfrak{z}}d\mathfrak{z}-(A+1)\overline{\int\frac{e^{\mathfrak{z}}}{V_0}\frac{dF}{d\mathfrak{z}}d\mathfrak{z}}\right]\right\}, \tag{1.6}$$

where $V_0=e^{\mathfrak{z}_0}$.

Significant for us in the present study is the particular case of A = -1. Assuming that A = -1, we get

$$\begin{cases} v(\sigma)=B\sigma e^{\sigma}, & \rho(\sigma)=\dfrac{1}{n^{2}\sigma(\sigma+1)}, \\ p(\sigma)=C_1-\dfrac{B^2e^{2\sigma}}{2n^2}, & M(\sigma)=\dfrac{\sqrt{2\sigma+1}}{\sigma+1}, \end{cases} \tag{1.7}$$

where $C_1 = C - B^2/2n^2$.

It is significant here that the formula of transition to the physical plane is also simplified:

(1.8)

$$Z=\frac{n^2}{2}\left\{\frac{e^{i\theta}}{v}\left[F(\mathfrak{z})-\overline{F(\mathfrak{z})}\right]+\frac{2}{B}\int\frac{V_0}{e^{\mathfrak{d}}}\frac{dF}{d\mathfrak{z}}d\mathfrak{z}\right\}.$$

§ 2

Let us introduce into our study the complex potential $V_\infty w(z)$ of a flow of an incompressible fluid, equilibrium at infinity, near a certain foil l. Here V_∞ represents the velocity at infinity, $z = x + iy$ - the plane coordinate of the flow of incompressible fluid.

In the neighborhood of the infinitely distant point for function $\zeta(z) = dw/dz$ we have the expansion in series

(2.1)

$$\zeta(z)=1+\frac{\Gamma^0}{2V_\infty\pi i}\frac{1}{z}+\frac{c_2}{z^2}+\ldots,$$

where Γ^0 represents circulation over the closed contour which encompasses foil l.

Let us use the condition

(2.2)

$$e^{\mathfrak{d}}=V_\infty\frac{dw}{dz},$$

as a result of which quantity e^{δ} acquires the sense of complex velocity, quantity $e^{\prime}$ - the sense of the velocity modulus V of the flow of incompressible fluid. Using this equality we establish the relationship between variables δ and ζ and the connection between variables $\mathfrak{z}$ and z. Function F can now be regarded as the function of the coordinate of the plane of the hodograph ζ or as the function of the plane coordinate of the flow of incompressible fluid z.

Let us determine function F(z) as follows:

(2.3) $$F(z) = kV_{\infty} w(z),$$

where k is a certain actual constant.

Then, for flow function ψ we get the representation

(2.4) $$\psi = \frac{kV_{\infty}}{e} \mathfrak{Im}\, w(z),$$

while the formula for transition to the plane of the gas flow acquires the form of

(2.5) $$Z = \frac{kn^2V_0}{B} z + \frac{kn^2V_{\infty}}{2}[w(z) - \overline{w(z)}]\frac{e^{i\theta}}{v}.$$

With the aid of the last formula we establish the connection

between the plane of the flow of incompressible fluid and the plane of the gas flow. Here an infinitely remote point on one plane corresponds to an infinitely remote point on the other plane. According to formulas (2.2), (2.5), and (1.7) we can plot in plane Z a gas flow which is equilibrium at infinity and corresponds to the original flow of incompressible fluid - also equilibrium at infinity.

On foil l the minimal part of the complex potential $V_\infty w(z)$ maintains a constant value. If we assume condition $\mathfrak{Im}\, V_\infty w(z) = 0$ on l, then on the basis of (2.4) and (2.5) at corresponding points on planes Z, determined according to formula

$$Z = \frac{kn^2 V_0}{B} z, \tag{2.6}$$

we will get the equality $\psi = 0$. This means that in plane Z the flow past foil L is a gas flow which is uniform at infinity, as with the original foil l. By the selection of constant k the scale factor in (2.6) can be returned to one, while foils l and L will be not only similar but also identical in value.

§ 3

Now let us determine the connection between pressure coefficient C^0_p at points on the foil l and pressure coefficient C_p at corresponding points on L.

On the basis of the third of formulas (1.7) we get

(3.1) $$p-p_\infty=\frac{B^2e^{2\sigma_\infty}}{2n^2}\left[1-\frac{e^{2\sigma}}{e^{2\sigma_\infty}}\right]=\frac{B^2e^{2\sigma_\infty}}{2n^2}\left[1-\frac{V^2}{V^2_\infty}\right].$$

Then, by using the Bernoulli equation for an incompressible fluid, from which it follows that

(3.2) $$C^0_p=1-\frac{V^2}{V^2_\infty},$$

and equalities

(3.3) $$B^2=\frac{v^2_\infty}{\sigma^2_\infty e^{2\sigma_\infty}},\quad n^2\sigma^2_\infty=\frac{\sqrt{1-M^2_\infty}}{\rho_\infty},$$

which emerge from formulas (1.7), we arrive at the unknown connection

(3.4) $$C_p=\frac{C^0_p}{\sqrt{1-M^2_\infty}}.$$

For the connection $v(\rho)$, determined by the first and second of

equations (1.7), the formula of the linearized theory thus becomes precise. As a result, from formula (3.4) we get, also in the form of a precise result, the relationship, known from the linearized theory, between the coefficient of the lifting force C_y of a foil in a gas flow and the coefficient of lifting force C^0_y for the same foil in a flow of an incompressible fluid:

$$C_y = \frac{C_y^0}{\sqrt{1-M_\infty^2}}. \tag{3.5}$$

Note that for the obtained relationship $v(\rho)$ we can give the elementary proof of the Zhukovskiy theorem of the lifting force acting on a foil. For this we should use, in place of relationships (3.5), (2.6), and the first and fourth from formulas (1.7), the Zhukovskiy theorem for an incompressible fluid and the relationship

$$\Gamma = kn^2(\sigma_\infty + 1)\Gamma^0 \tag{3.6}$$

between circulation of velocity Γ^0 over the closed contour encompassed by foil l and circulation of the velocity Γ over the closed contour encompassed by foil L. Relationship (3.6) follows directly from the expression for the velocity potential (1.3) using an expansion in series of function $\zeta(z)$.

§ 4

Freedom in the selection of constants n, B, and C_1 can be used for approximations of the laws of the adiabatic flow of an ideal gas or the laws of other barotropic flows which interest us. Such approximations can be plotted by different methods. Here we use the method discussed in [1], according to which the approximation is done for the values of parameters at a certain characteristic point in the flow. As such a characteristic point we use an infinitely distant point, representing the assigned values of gas parameters in it as ρ_∞, p_∞, v_∞, and M_∞.

The three arbitrary constants n, B, and C_1 are obviously sufficient to satisfy equalities $v(\rho_\infty) = v_\infty$, $p(\rho_\infty) = p_\infty$, $M(\rho_\infty) = M_\infty$. Here the equality $K(\rho_\infty) = K_\infty$ is satisfied, where K_∞ represents the known value of the Chaplygin function at the characteristic point and, in addition, at an assigned point (ρ_∞, p_∞) on plane ρp we have the first order of contact of the assigned and approximating curves $p(\rho)$.

In Figs. 1-4 dashed lines 3 represent approximations of the laws of the adiabatic flow of an ideal gas (solid lines 1) for $M_\infty = 0.7$.

The approximating dependences are determined by means of

equation (1.7) and (1.2). Here

(4.1)
$$n=\frac{1-\sqrt{1-M_\infty^2}}{[\rho_\infty\sqrt{1-M_\infty^2}]^{1/2}},$$

while the formulas for pressure and velocity, after selection of constants B and C_1, acquire the form of

(4.2)
$$\frac{p}{p_\infty}=1-\frac{\gamma M_\infty^2}{2\sqrt{1-M_\infty^2}}\left[\frac{e^{2\sigma}}{e^{2\sigma_\infty}}-1\right],\quad \frac{v}{v_\infty}=\frac{\sigma e^{\sigma}}{\sigma_\infty e^{\sigma_\infty}},$$

where

(4.3)
$$\sigma_\infty=\frac{\sqrt{1-M_\infty^2}}{1-\sqrt{1-M_\infty^2}}$$

and γ is the adiabatic exponent. We will assume that quantity ρ in formulas (1.7) and (4.1) is a dimensionless quantity representing the ratio of gas density to density at the stagnation point of an adiabatic flow of ideal gas.

For comparison in Figs. 1-4 the curves plotted from the approximate Chaplygin method (K = const) are shown as thin solid lines 2. Here the approximation is done by the same method [1] for M_∞ = 0.7.

In both methods, for the assigned value $\rho = \rho_\infty$, the Poisson

adiabatic curve is approximated accurate to within small values of $(\rho - \rho_\infty)^2$, although within a broad range of parameters change the approximate Chaplygin method has one advantage: Its laws, as we see from the curves, are in much better agreement with the laws of the adiabatic flow of ideal gas.

§ 5

Let us point out certain conditions which limit the applicability of this theory.

From the formula for M it follows that $\sigma \geqslant -1/2$. Otherwise, we get imaginary values for the M number. On the other hand, when $-1 < \sigma < 0$, quantity ρ, according to the second of formulas (1.7), acquires negative values. Consequently, for physical considerations, variable σ 0 cannot be negative: $\sigma \geqslant 0$. Since on the basis of (2.2) and (4.3)

(5.1)
$$\sigma = \ln|\zeta| + \frac{\sqrt{1 - M_\infty^2}}{1 - \sqrt{1 - M_\infty^2}},$$

we come to the condition

(5.2)
$$\ln|\zeta| \geqslant \frac{\sqrt{1 - M_\infty^2}}{\sqrt{1 - M_\infty^2} - 1}.$$

From this condition it is specifically apparent that velocity should not revert to zero at a single point in the original flow of the incompressible fluid. This fact naturally imposes serious limitations on the shape of the foil and its position in the flow.

The second limitation is related to the formula for pressure (4.2). This formula looses its physical sense at values σ to which negative pressure values correspond. From the condition $p \geqslant 0$ we get

$$|\zeta| \leqslant \sqrt{\frac{2\sqrt{1-M_\infty^2}}{\gamma M_\infty^2}+1}. \tag{5.3}$$

Obviously this inequality also imposes limitations on the shape of the foil and its position in the flow.

§ 6

When $M_\infty \to 0$ the solution plotted for a compressible flow moves toward the solution for the corresponding problem of an incompressible fluid.

To be convinced of this, let us transform the formulas for v and Z, explicitly isolating the dependence on parameter M_∞:

(6.1)
$$\frac{v}{v_\infty}=\frac{V}{V_\infty}\frac{\ln|\zeta|(1-\sqrt{1-M_\infty^2})+\sqrt{1-M_\infty^2}}{\sqrt{1-M_\infty^2}},$$

(6.2)
$$Z=z+\frac{1}{2}\frac{1-\sqrt{1-M_\infty^2}}{\ln|\zeta|(1-\sqrt{1-M_\infty^2})+\sqrt{1-M_\infty^2}}\frac{w(z)-\overline{w(z)}}{\zeta(z)}.$$

Used in the transformation process were condition (2.2) and equality (5.1). Quantity k is determined from the condition of equality to one of the scale factors in (2.6), i.e., we assume

(6.3)
$$k=\frac{B}{n^2V_0}.$$

Formulas (6.1) and (6.2) determine for an assigned M_∞ the field of the velocities of the gas flow from the known field of flow velocities of an incompressible fluid (at corresponding points on plane z and plane Z the angles of slope of the velocity vectors according to condition (2.2) are the same). From (6.1) and (6.2) it is apparent that when $M_\infty \to 0$, for each fixed value z we get $Z \to z$, $v/v_\infty \to V/V_\infty$. Thus, when $M_\infty \to 0$ the field of velocities of the gas flow near the foil become similar to the field of velocities of the original flow of incompressible fluid.

It is also simple to verify the following transitions to the limit when $M_\infty \to 0$: $\rho \to 1$, $K \to 1$, $\Gamma/v_\infty \to \Gamma^0/V_\infty$. Specifically, the last transition to the limit follows directly from relationship

(6.4)
$$\frac{\Gamma}{v_\infty}=\frac{1}{\sqrt{1-M_\infty^2}}\frac{\Gamma^\bullet}{V_\infty},$$

which emerges after simple transformations from relationship (3.6) indicated above.

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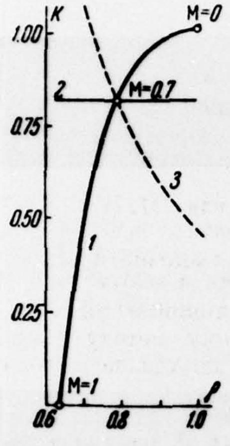

Fig. 1.

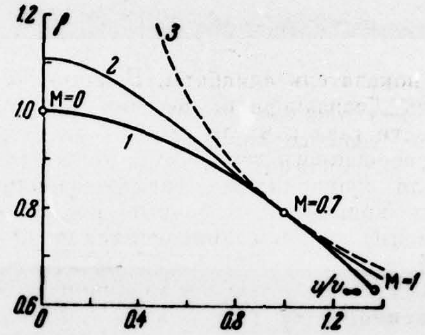

Fig. 2.

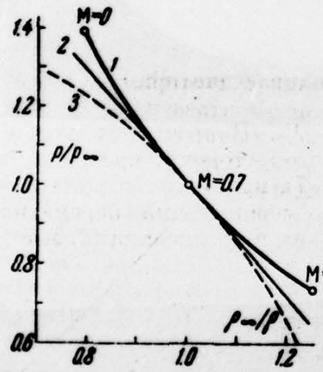

Fig. 3.

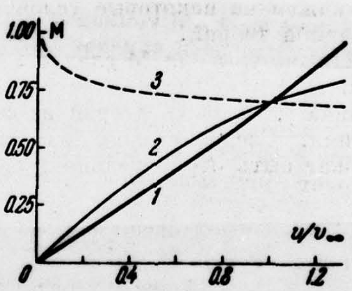

Fig. 4.

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